Vector Meson Photoproduction with an Effective Lagrangian in the Quark Model I: Formalism

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Abstract

A quark model approach to the photoproduction of vector mesons off nucleons is proposed. Its starting point is an effective Lagrangian of the interaction between the vector meson and the quarks inside the baryon. This paper develops the formalism and highlights the dynamical roles of s-channel resonances. It is demonstrated that the proposed approach is ideal to support searching for the missing resonances in the vector meson photoproduction experiments proposed at TJNAF.

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1. Introduction

The newly established electron and photon facilities have made it possible to investigate the photoproduction of vector mesons, such as ω and ρ, on the nucleon with much better energy and angular resolutions. Such experiments have already been carried out at ELSA[1], and will be performed at TJNAF[2] in the near future. The data from these measurements should be able to help us resolve the puzzle of the so-called "missing resonance"[3, 4]. A better understanding of the reaction mechanism is therefore crucial in order to extract resonance information from vector meson photoproduction observables. Historically, one of the attempts to understand ρ photoproduction in the reaction $\gamma p \to \pi^+\pi^- p$ has lead to the so-called "Söding Model"[5], which explains low-energy ρ^0 production mainly through t-channel exchange. Although the Söding Model has had some success in reproducing the data in certain kinematic regions it cannot be used to study the dynamical role of the s-channel resonances in vector meson photoproduction. A more systematic approach that includes the t-channel exchange terms along with the s- and u-channel resonances is needed to meet the challenge of new high precision data and serve the purpose of searching for "missing resonances".

The constituent quark model approach has been shown to very successfully reproduce data in the area of pseudoscalar meson photoproduction [6] with a minimal number of

free parameters since it introduces the quark degrees of freedom directly into the reaction mechanism. More importantly, it allows to study the internal structure of the contributing s-channel resonances directly in connection with the reaction mechanism. It is therefore natural to extend the quark model approach to vector meson photoproduction in the resonance region, the development of the formalism for that will be the focus of this paper.

A major difference for vector meson production from the pseudoscalar meson case in the quark model is that the interaction between the vector mesons and the quarks inside the baryon is largely unknown. Although phenomenological models have been developed to evaluate baryon resonance decays into a nucleon and a vector meson, such as the quark pair creaction model [7] or the $^3P_0$ model, these approaches are unsuitable for the description of vector meson photoproductions. This is due to the fact that they only yield transition amplitudes for s-channel resonances, but contain no information on how to derive the non-resonant terms in the u- and t-channels. Therefore, we choose an effective Lagrangian here as our starting point that satisfies the fundamental symmetries and determines the transition amplitudes not only in the s-channel but also in the t- and u- channels.

Even though the effective Lagrangians are different for pseudoscalar and vector meson photoproductions, the implementation follows the same guidelines. The transition amplitudes for each resonance in the s-channel below 2 GeV will be included explicitly, while the resonances above 2 GeV for a given quantum number n in the harmonic oscillator basis of the quark model are treated as degenerate, so that their transition amplitudes can be written in a compact form. Similarly, the excited resonances in the u-channel are assumed to be degenerate as well. Only the mass splitting between the spin $1/2$ and spin $3/2$ resonances with $n = 0$ in the harmonic oscillator basis, such as the splitting between nucleon and Δ resonance, is found to be significant, thus, the transition amplitudes for the spin $3/2$ resonance with $n = 0$ in the u-channel will be included separately.

The effective Lagrangian employed here generates not only the s- and u-channel exchanges but also a t-channel term containing the vector meson. For charged vector mesons gauge invariance also mandates a seagull term. An open question in this approach is whether additional t-channel contribution, such as the Pomeron exchange, responsible for the diffractive behavior in the small t region is needed. Previous studies in the pseudoscalar sector suggest that additional t-channel exchanges were not required to describe the data. This could be understood via application of the duality hypothesis [14]. However, whether it is applicable for vector meson photoproduction is not clear at present, this will be investigated further by comparing the numerical results of the model with experimental data.

In Section 2, we briefly discuss some of the observables used in our approach, which have been developed extensively in Ref. [8]. The effective Lagrangian for the quark-meson interaction is presented in Section 3, along with detail expressions for the the s-, u- and t-channel contributions arising from this Lagrangian. Finally, conclusions will be presented in the Section 4.

2. Observables and Helicity Amplitudes

Before presenting our quark model approach we introduce some general features of vector meson photoproduction on the nucleon. The basic amplitude $\mathcal{F}$ for $\gamma + N \to V + N$ is defined as

$$\mathcal{F} = \langle \mathbf{q}\lambda_V \lambda_2 | T | \mathbf{k}\lambda\lambda_1 \rangle, \tag{1}$$

where $\mathbf{k}$ and $\mathbf{q}$ are the momenta of the incoming photon and outgoing vector meson. The helicity states are denoted by $\lambda = \pm 1$ for the incident photon, $\lambda_V = 0, \pm 1$ for the outgoing vector meson, and $\lambda_1 = \pm 1/2$,$\lambda_2 = \pm 1/2$ for the initial and final state nucleons, respectively. Following Ref. [8], the amplitude $\mathcal{F}$ can be expressed as a 6×4 matrix in the helicity space,

$$\mathcal{F} = \begin{pmatrix} H_{21} & H_{11} & H_{3-1} & -H_{4-1} \\ H_{41} & H_{31} & -H_{1-1} & H_{2-1} \\ H_{20} & H_{10} & -H_{30} & H_{40} \\ H_{40} & H_{30} & H_{10} & -H_{20} \\ H_{2-1} & H_{1-1} & H_{31} & -H_{41} \\ H_{4-1} & H_{3-1} & -H_{11} & H_{21} \end{pmatrix}. \tag{2}$$

Because of the parity conservation,

$$\langle \mathbf{q}\lambda_V \lambda_2 | T | \mathbf{k}\lambda\lambda_1 \rangle = (-1)^{\Lambda_f - \Lambda_i} \langle \mathbf{q} - \lambda_V - \lambda_2 | T | \mathbf{k} - \lambda - \lambda_1 \rangle, \tag{3}$$

where $\Lambda_f = \lambda - \lambda_1$ and $\Lambda_i = \lambda_V - \lambda_2$ in the Jacob-Wick(JW) convention, the $H_{a\lambda_V}(\theta)$ in Eq.(2) reduces to 12 independent complex helicity amplitudes:

$$\begin{aligned} H_{1\lambda_V} &= \langle \lambda_V, \lambda_2 = +1/2 | T | \lambda = 1, \lambda_1 = -1/2 \rangle \\ H_{2\lambda_V} &= \langle \lambda_V, \lambda_2 = +1/2 | T | \lambda = 1, \lambda_1 = +1/2 \rangle \\ H_{3\lambda_V} &= \langle \lambda_V, \lambda_2 = -1/2 | T | \lambda = 1, \lambda_1 = -1/2 \rangle \\ H_{4\lambda_V} &= \langle \lambda_V, \lambda_2 = -1/2 | T | \lambda = 1, \lambda_1 = +1/2 \rangle. \end{aligned} \tag{4}$$

Each experimental observable Ω can be written in the general *bilinear helicity product*(BHP) form,

$$\begin{aligned} \check{\Omega}^{\alpha\beta} &= \Omega^{\alpha\beta} \mathcal{T}(\theta) \\ &= \pm \frac{1}{2} \langle H | \Gamma^\alpha \omega^\beta | H \rangle \\ &= \pm \frac{1}{2} \sum_{a,b,\lambda_V,\lambda'_V} H^*_{a\lambda_V} \Gamma^\alpha_{ab} \omega^\beta_{\lambda_V \lambda'_V} H_{b\lambda'_V}. \end{aligned} \tag{5}$$

For example, the differential cross section operator is given by:

$$\begin{aligned} \check{\Omega}^{\alpha=1,\beta=1} &= \mathcal{T}(\theta) \\ &= \frac{1}{2} \langle H | \boxed{\Gamma^1} \boxed{\omega^1} | H \rangle \\ &= \frac{1}{2} \sum_{a=1}^{4} \sum_{\lambda_V = 0, \pm 1} |H_{a\lambda_V}|^2, \end{aligned} \tag{6}$$

where the box frames denote the diagonal structure of the matrices. The Γ and ω matrices labeled by different α and β correspond to different spin observables. With the phase space factor, the differential cross section has the expression,

$$
\begin{aligned}
\frac{d\sigma}{d\Omega_{c.m.}} &= (P.S.factor)\mathcal{T}(\theta) \\
&= \frac{\alpha_e \omega_m (E_f + M_N)(E_i + M_N)}{8\pi s} |\mathbf{q}| \frac{1}{2} \sum_{a=1}^{4} \sum_{\lambda_V = 0, \pm 1} |H_{a\lambda_V}|^2 \qquad (7)
\end{aligned}
$$

in the center of mass frame, where $\sqrt{s}$ is the total energy of the system, μ and M_N represent the masses of the outgoing meson and nucleon, and ω, ω_m denote the energy of the photon and meson, respectively.

These helicity amplitudes are usually related to the density matrix elements ρ_{ik} [10], which are measured by the experiments [11]. They are defined as:

$$
\begin{aligned}
\rho^0_{ik} &= \frac{1}{A} \sum_{\lambda\lambda_2\lambda_1} H_{\lambda_{V_i}\lambda_2, \lambda\lambda_1} H^*_{\lambda_{V_k}\lambda_2, \lambda\lambda_1}, \\
\rho^1_{ik} &= \frac{1}{A} \sum_{\lambda\lambda_2\lambda_1} H_{\lambda_{V_i}\lambda_2, -\lambda\lambda_1} H^*_{\lambda_{V_k}\lambda_2, \lambda\lambda_1}, \\
\rho^2_{ik} &= \frac{i}{A} \sum_{\lambda\lambda_2\lambda_1} \lambda H_{\lambda_{V_i}\lambda_2, -\lambda\lambda_1} H^*_{\lambda_{V_k}\lambda_2, \lambda\lambda_1}, \\
\rho^3_{ik} &= \frac{i}{A} \sum_{\lambda\lambda_2\lambda_1} \lambda H_{\lambda_{V_i}\lambda_2, \lambda\lambda_1} H^*_{\lambda_{V_k}\lambda_2, \lambda\lambda_1}, \qquad (8)
\end{aligned}
$$

where

$$
A = \sum_{\lambda_{V_i}\lambda\lambda_2\lambda_1} H_{\lambda_{V_i}\lambda_2, \lambda\lambda_1} H^*_{\lambda_{V_k}\lambda_2, \lambda\lambda_1}, \qquad (9)
$$

where ρ_{ik} stands for $\rho_{\lambda_{V_i}\lambda_{V_k}}$, and λ_{V_i}, λ_{V_k} denote the helicity of the produced vector mesons.

For example, the angular distribution for ρ^0 decaying into $\pi^+\pi^-$ produced by linearly polarized photons can be expressed in terms of nine independent measurable spin-density matrix elements

$$
\begin{aligned}
W(cos\theta, \phi, \Phi) &= \frac{3}{4\pi}[\frac{1}{2}(1-\rho^0_{00}) + \frac{1}{2}(3\rho^0_{00} - 1)cos^2\theta - \sqrt{2}Re\rho^0_{10} sin2\theta cos\phi \\
&\quad -\rho^0_{1-1} sin^2\theta cos2\phi - P_\gamma cos2\Phi(\rho^1_{11} sin^2\theta + \rho^1_{00} cos^2\theta \\
&\quad -\sqrt{2}Re\rho^1_{10} sin2\theta cos\phi - \rho^1_{1-1} sin^2\theta cos2\phi) \\
&\quad -P_\gamma sin2\phi(\sqrt{2}Im\rho^2_{10} sin2\theta sin\phi + Im\rho^2_{1-1} sin^2\theta sin2\phi)], \qquad (10)
\end{aligned}
$$

where P_γ is the degree of the linear polarization of the photon, Φ is the angle of the photon electric polarization vector with respect to the production plane measured in the c.m. system, and θ and ϕ are the polar and azimuthal angles of the π^+ which is produced by the ρ^0 decay in the ρ^0 rest frame.

3. Quark Model Approach for Vector Meson Photoproduction

The starting point of the quark model approach is the effective Lagrangian,

$$L_{eff} = -\overline{\psi}\gamma_\mu p^\mu \psi + \overline{\psi}\gamma_\mu e_q A^\mu \psi + \overline{\psi}(a\gamma_\mu + \frac{ib\sigma_{\mu\nu}q^\nu}{2m_q})\phi_m^\mu \psi, \tag{11}$$

where the quark field ψ is expressed as

$$\psi = \begin{pmatrix} \psi(u) \\ \psi(d) \\ \psi(s) \end{pmatrix}, \tag{12}$$

and the meson field ϕ_m^μ is a 3$\otimes$3 matrix,

$$\phi_m = \begin{pmatrix} \frac{1}{\sqrt{2}}\rho^0 + \frac{1}{\sqrt{2}}\omega & \rho^+ & K^{*+} \\ \rho^- & -\frac{1}{\sqrt{2}}\rho^0 + \frac{1}{\sqrt{2}}\omega & K^{*0} \\ K^{*-} & \overline{K}^{*0} & \phi \end{pmatrix} \tag{13}$$

in which the vector mesons are treated as point-like particles. At tree level, the transition matrix element based on the effective Lagrangian in Eq.(11) can be written as the sum of contributions from the s-, u- and t- channels,

$$M_{fi} = M_{fi}^s + M_{fi}^u + M_{fi}^t, \tag{14}$$

where the s- and u-channel contributions in Eq.(14) have the following form,

$$\begin{aligned} M_{fi}^s + M_{fi}^u &= \sum_j \langle N_f|H_m|N_j\rangle\langle N_j|\frac{1}{E_i+\omega-E_j}H_e|N_i\rangle \\ &\quad + \sum_j \langle N_f|H_e\frac{1}{E_i-\omega_m-E_j}|N_j\rangle\langle N_j|H_m|N_i\rangle, \end{aligned} \tag{15}$$

where the electromagnetic coupling vertex is

$$H_e = -\overline{\psi}\gamma_\mu e_q A^\mu \psi, \tag{16}$$

and the quark-meson coupling vertex is

$$H_m = -\overline{\psi}(a\gamma_\mu + \frac{ib\sigma_{\mu\nu}q^\nu}{2m_q})\phi_m^\mu \psi, \tag{17}$$

where m_q in is the quark mass and the constants a and b in Eq.(11) and (17) are the vector and tensor coupling constants, which will be treated as free parameters in our approach. The initial and final states of the nucleon are denoted by $|N_i\rangle$ and $|N_f\rangle$, respectively, and $|N_j\rangle$ is the intermediate resonance state while E_i and E_j are the energies of the inital nucleon and the intermediate resonance.

An important test of the transition matrix elements

$$M_{fi} = \langle\lambda_2|J_{\mu\nu}\epsilon^\mu\epsilon_m^\nu|\lambda_1\rangle, \tag{18}$$

would be gauge invariance:

$$\langle\lambda_2|J_{\mu\nu}k^\mu|\lambda_1\rangle = \langle\lambda_2|J_{\mu\nu}q_m^\nu|\lambda_1\rangle = 0, \tag{19}$$

where ϵ_m^μ, ϵ^ν are the polarization vectors of the vector mesons and photons. However, we find that the condition $\langle\lambda_2|J_{\mu\nu}q_m^\nu|\lambda_1\rangle = 0$ is not satisfied for the t-channel vector meson exchange term,

$$M_{fi}^t = a\langle N_f|\sum_l \frac{e_m}{2q\cdot k}\{2q\cdot\epsilon\gamma\cdot\epsilon_m - \gamma\cdot q\epsilon\cdot\epsilon_m + k\cdot\epsilon_m\gamma\cdot\epsilon\}e^{i(\mathbf{k}-\mathbf{q})\cdot\mathbf{r}_l}|N_i\rangle, \tag{20}$$

based on the Feynman rules for the photon-vector meson coupling, and the relation

$$\langle N_f|\gamma\cdot(q-k)e^{i(\mathbf{k}-\mathbf{q})\cdot\mathbf{r}_l}|N_i\rangle = 0. \tag{21}$$

To remedy this problem, we add a gauge fixing term, so that

$$M_{fi}^t = a\langle N_f|\sum_l \frac{e_m}{2q\cdot k}2\{q\cdot\epsilon\gamma\cdot\epsilon_m - \gamma\cdot q\epsilon\cdot\epsilon_m + k\cdot\epsilon_m\gamma\cdot\epsilon\}e^{i(\mathbf{k}-\mathbf{q})\cdot\mathbf{r}_l}|N_i\rangle. \tag{22}$$

The techniques of deriving the transition amplitudes have been developed for Compton scattering [13]. Follow the same procedure as given in Eq.(14) of Ref. [6], we can divide the photon interaction into two parts and the contributions from the s- and u- channel can be rewritten as:

$$\begin{aligned} M_{fi}^{s+u} &= i\langle N_f|[g_e, H_m]|N_i\rangle \\ &\quad + i\omega\sum_j\langle N_f|H_m|N_j\rangle\langle N_j|\frac{1}{E_i+\omega-E_j}h_e|N_i\rangle \\ &\quad + i\omega\sum_j\langle N_f|h_e\frac{1}{E_i-\omega_m-E_j}|N_j\rangle\langle N_j|H_m|N_i\rangle, \end{aligned} \tag{23}$$

where

$$g_e = \sum_l e_l\mathbf{r}_l\cdot\epsilon e^{i\mathbf{k}\cdot\mathbf{r}_l}, \tag{24}$$

$$h_e = \sum_l e_l\mathbf{r}_l\cdot\epsilon(1-\boldsymbol{\alpha}\cdot\hat{\mathbf{k}})e^{i\mathbf{k}\cdot\mathbf{r}_l} \tag{25}$$

$$\hat{\mathbf{k}} = \frac{\mathbf{k}}{\omega}. \tag{26}$$

The first term in Eq.(23) can be identified as a seagull term; it is proportional to the charge of the outgoing vector meson. The second and third term in Eq.(23) represents the s- and u-channel contributions. Adopting the same strategy as in the pseudoscalar case, we include a complete set of helicity amplitudes for each of the s-channel resonances below

2GeV in the $SU(6) \otimes O(3)$ symmetry limit. The resonances above 2GeV are treated as degenerate in order to express each contribution from all resonances with quantum number n in a compact form. The contributions from the resonances with the largest spin for a given quantum number n were found to be the most important as the energy increases [6]. This corresponds to spin $J = n + 1/2$ with $I = 1/2$ for the reactions $\gamma N \to K^*\Lambda$ and $\gamma N \to \omega N$, and $J = n + 3/2$ with $I = 3/2$ for the reactions $\gamma N \to K^*\Sigma$ and $\gamma N \to \rho N$.

Similar to the pseudoscalar case, the contributions from the u-channel resonances are divided into two parts as well. The first part contains the resonances with the quantum number $n = 0$, which include the spin 1/2 states, such as the Λ, Σ and the nucleons, and the spin 3/2 resonances, such as the Σ^* in K^* photoproduction and $\Delta(1232)$ resonance in ρ photoproduction. Because the mass splitting between the spin 1/2 and spin 3/2 resonances for $n = 0$ is significant, they have to be treated separately. The transition amplitudes for these u-channel resonances will also be written in terms of the helicity amplitudes. The second part comes from the excited resonances with quantum number $n \geq 1$. As the contributions from the u-channel resonances are not sensitive to the precise mass positions, they can be treated as degenerate as well, so that the contributions from these resonances can again be written in a compact form.

3.1. The seagull term

The transition amplitude is divided into the transverse and longitudinal amplitudes according to the polarization of the outgoing vector mesons. The longitudinal polarization vector for a vector meson with mass μ and momentum $\mathbf{q}$ is,

$$\epsilon_L^\mu = \frac{1}{\mu} \begin{pmatrix} |\mathbf{q}| \\ \omega_m \frac{\mathbf{q}}{|\mathbf{q}|} \end{pmatrix} \tag{27}$$

where $\omega_m = \sqrt{\mathbf{q}^2 + \mu^2}$ is the energy of the outgoing vector mesons. Thus, the longitudinal interaction at the quark-meson vertex can be written as

$$H_m^L = \epsilon_L^\mu J_\mu = \epsilon_0 J_0 - \epsilon_3 J_3 \tag{28}$$

where ϵ_3 corresponds to the direction of the momentum $\mathbf{q}$. The transition amplitudes of the s- and u-channel for the longitudinal quark-meson coupling become,

$$\begin{aligned} M_{fi}^{s+u}(L) &= i\langle N_f|[g_e, H_m^L]|N_i\rangle \\ &\quad -i\omega\langle N_f|[h_e, \frac{\epsilon_3}{q_3}J_0]|N_i\rangle \\ &\quad +i\omega\sum_j\langle N_f|(\epsilon_0 - \frac{\omega_m}{q_3}\epsilon_3)J_0|N_j\rangle\langle N_j|\frac{1}{E_i+\omega-E_j}h_e|N_i\rangle \\ &\quad +i\omega\sum_j\langle N_f|h_e\frac{1}{E_i-\omega_m-E_j}|N_j\rangle\langle N_j|(\epsilon_0 - \frac{\omega_m}{q_3}\epsilon_3)J_0|N_i\rangle,) \end{aligned} \tag{29}$$

where the first two terms are seagull terms which are cancelled by similar terms in the t-channel. The corresponding expressions for the t-channel amplitudes are given in the appendix; the last two terms will be discussed in the following sections.

The nonrelativistic expansion of the transverse meson quark interaction vertex gives,

$$H_m^T = \sum_l \{ i\frac{b'}{2m_q}\boldsymbol{\sigma}_l \cdot (\mathbf{q}\times\boldsymbol{\epsilon}_v) + a\mathbf{A}\cdot\boldsymbol{\epsilon}_v + \frac{a}{2\mu_q}\mathbf{p}'_l\cdot\boldsymbol{\epsilon}_v\}\hat{I}_l e^{-i\mathbf{q}\cdot\mathbf{r}_l} \tag{30}$$

where $b' = b - a$ and

$$\hat{I}_l = \begin{cases} a_l^\dagger(s)a_l(u) & \text{for} & K^{*+} \\ a_l^\dagger(s)a_l(d) & \text{for} & K^{*0} \\ a_l^\dagger(d)a_l(u) & \text{for} & \rho^+ \\ -\frac{1}{\sqrt{2}}(a_l^\dagger(u)a_l(u) - a_l^\dagger(d)a_l(d)) & \text{for} & \rho^0 \\ 1 & \text{for} & \omega \end{cases} \tag{31}$$

The vector $\mathbf{A}$ has the general form,

$$\mathbf{A} = \frac{\mathbf{P}_f}{E_f + M_f} + \frac{\mathbf{P}_i}{E_i + M_i}, \tag{32}$$

which comes from the center-mass motion of the quark system. In the s- and u-channel, $\mathbf{A}$ has the following expression for different channels,

$$s - channel: \qquad \mathbf{A} = -\frac{\mathbf{q}}{E_f + M_f}, \tag{33}$$

$$u - channel: \qquad \mathbf{A} = -(\frac{1}{E_f + M_f} + \frac{1}{E_i + M_i})\mathbf{k} - \frac{1}{E_f + M_f}\mathbf{q}. \tag{34}$$

The transverse transition amplitude for the s- and u-channel is,

$$\begin{aligned} M_{fi}^{s+u}(T) &= i\langle N_f|[g_e, H_m^T]|N_i\rangle \\ &+ i\omega\sum_j \langle N_f|H_m^T|N_j\rangle\langle N_j|\frac{1}{E_i + \omega - E_j}h_e|N_i\rangle \\ &+ i\omega\sum_j \langle N_f|h_e\frac{1}{E_i - \omega_m - E_j}|N_j\rangle\langle N_j|H_m^T|N_i\rangle \end{aligned} \tag{35}$$

The nonrelativistic expansion of the first term gives,

$$\begin{aligned} M_{fi}^{seagull}(T) &= -ig_3^s a e_m\{g_v\langle N_f|\{\mathbf{A}\cdot\boldsymbol{\epsilon}_v, \mathbf{R}\cdot\boldsymbol{\epsilon}\}|N_i\rangle \\ &+ g_v\langle N_f|\sum_l\{\frac{1}{2m_l}\mathbf{p}'_l\cdot\boldsymbol{\epsilon}_v, \mathbf{r}'_l\cdot\boldsymbol{\epsilon}\}|N_i\rangle \\ &- \langle N_f|\{\frac{1}{E+M} + \frac{1}{2m_q}\}\boldsymbol{\sigma}\cdot(\boldsymbol{\epsilon}_v\times\boldsymbol{\epsilon})|N_i\rangle\}e^{-\frac{(\mathbf{k}-\mathbf{q})^2}{6\alpha^2}} \end{aligned} \tag{36}$$

where $\{A, B\} = AB + BA$ is the anti-commutation operator. $\mathbf{R}$ is the coordinate of the center mass motion of the three quark system and $\mathbf{q}'_l$, $\mathbf{r}'_l$ denote the internal momentum and coordinate of the lth quark.

The seagull terms in the transverse transitions are proportional to the charge of the outgoing mesons and, therefore, vanish in neutral vector meson photoproduction.

3.2. U-channel transition amplitudes

The last term in Eq.(29) is the longitudinal transition amplitude in the u-channel. We find

$$
\begin{aligned}
M^u_{fi}(L) &= i\omega\sum_j \langle N_f|h_e\frac{1}{E_i-\omega_m-E_j}|N_j\rangle\langle N_j|-\frac{\mu}{|\mathbf{q}|}J_0|N_i\rangle \\
&= (M^u_3+M^u_2)e^{-\frac{\mathbf{q}^2+\mathbf{k}^2}{6\alpha^2}} \qquad (37)
\end{aligned}
$$

in the harmonic oscillator basis, where

$$
\begin{aligned}
M^u_3 &= g^u_3\frac{a\mu}{|\mathbf{q}|}\{\frac{i}{2m_q}\boldsymbol{\sigma}\cdot(\boldsymbol{\epsilon}\times\mathbf{k})F^0(\frac{\mathbf{k}\cdot\mathbf{q}}{3\alpha^2},P_f\cdot k) \\
&\quad -g_v\frac{\omega}{3\alpha^2}\mathbf{q}\cdot\boldsymbol{\epsilon}\,F^1(\frac{\mathbf{k}\cdot\mathbf{q}}{3\alpha^2},P_f\cdot k)\}, \qquad (38)
\end{aligned}
$$

which corresponds to incoming photons and outgoing vector mesons being absorbed and emitted by the same quark, and

$$
\begin{aligned}
M^u_2 &= g^u_2\frac{a\mu}{|\mathbf{q}|}\{g'_a\frac{i}{2m_q}\boldsymbol{\sigma}\cdot(\boldsymbol{\epsilon}\times\mathbf{k})F^0(-\frac{\mathbf{k}\cdot\mathbf{q}}{6\alpha^2},P_f\cdot k) \\
&\quad +g'_v\frac{\omega}{6\alpha^2}\mathbf{q}\cdot\boldsymbol{\epsilon}\,F^1(-\frac{\mathbf{k}\cdot\mathbf{q}}{6\alpha^2},P_f\cdot k)\} \qquad (39)
\end{aligned}
$$

in which the incoming photons and outgoing vector mesons are absorbed and emitted by different quarks. P_f in Eq.(38) and Eq.(39) denotes the four momentum of the final state nucleon. The function F in Eq.(38) and Eq.(39) is defined as,

$$
F^l(x,y)=\sum_{n\geq l}\frac{M_n}{(n-l)!(y+n\delta M^2)}x^{n-l}, \qquad (40)
$$

where $n\delta M^2=(M_n^2-M_f^2)/2$ represents the average mass difference between the ground state and excited states with the total excitation quantum number n in the harmonic oscillator basis. The parameter α^2 in the above equation is commonly used in the quark model and is related to the harmonic oscillator strength.

Similarly, the transverse transition in the u-channel is given by,

$$
\begin{aligned}
M^u_{fi}(T) &= i\omega\sum_j \langle N_f|h_e\frac{1}{E_i-\omega_m-E_j}|N_j\rangle\langle N_j|H^T_m|N_i\rangle \\
&= (M^u_3+M^u_2)e^{-\frac{\mathbf{q}^2+\mathbf{k}^2}{6\alpha^2}} \qquad (41)
\end{aligned}
$$

where

$$
\begin{aligned}
M^u_3/g^u_3 &= \frac{b'}{4m_q^2}\{g_v(\boldsymbol{\epsilon}\times\mathbf{k})\cdot(\mathbf{q}\times\boldsymbol{\epsilon}_v)+i\boldsymbol{\sigma}\cdot(\boldsymbol{\epsilon}\times\mathbf{k})\times(\mathbf{q}\times\boldsymbol{\epsilon}_v)\}F^0(\frac{\mathbf{k}\cdot\mathbf{q}}{3\alpha^2},P_f\cdot k) \\
&\quad -\frac{ia}{2m_q}\boldsymbol{\sigma}\cdot(\boldsymbol{\epsilon}\times\mathbf{k})\mathbf{A}\cdot\boldsymbol{\epsilon}_vF^0(\frac{\mathbf{k}\cdot\mathbf{q}}{3\alpha^2},P_f\cdot k)
\end{aligned}
$$

$$
\begin{aligned}
&+\{\frac{ia}{12m_q^2}\boldsymbol{\sigma}\cdot(\boldsymbol{\epsilon}\times\mathbf{k})\boldsymbol{\epsilon}_v\cdot\mathbf{k}+\frac{ib'\omega}{6m_q\alpha^2}\boldsymbol{\sigma}\cdot(\mathbf{q}\times\boldsymbol{\epsilon}_v)\boldsymbol{\epsilon}\cdot\mathbf{q}\\
&+g_v\frac{a\omega}{3\alpha^2}\boldsymbol{\epsilon}\cdot\mathbf{q}\mathbf{A}\cdot\boldsymbol{\epsilon}_v-g_v\frac{a\omega}{6m_q}\boldsymbol{\epsilon}\cdot\boldsymbol{\epsilon}_v\}F^1(\frac{\mathbf{k}\cdot\mathbf{q}}{3\alpha^2},P_f\cdot k)\\
&-g_v\frac{a\omega}{18m_q\alpha^2}\boldsymbol{\epsilon}_v\cdot\mathbf{k}\boldsymbol{\epsilon}\cdot\mathbf{q}F^2(\frac{\mathbf{k}\cdot\mathbf{q}}{3\alpha^2},P_f\cdot k)
\end{aligned}
\tag{42}
$$

and

$$
\begin{aligned}
M_2^u/g_2^u\ =\ &\frac{b'}{4m_q^2}\{g_v'(\boldsymbol{\epsilon}\times\mathbf{k})\cdot(\mathbf{q}\times\boldsymbol{\epsilon}_v)\\
&+ig_a'\boldsymbol{\sigma}\cdot(\boldsymbol{\epsilon}\times\mathbf{k})\times(\mathbf{q}\times\boldsymbol{\epsilon}_v)\}F^0(-\frac{\mathbf{k}\cdot\mathbf{q}}{6\alpha^2},P_f\cdot k)\\
&-\frac{ia}{2m_q}\boldsymbol{\sigma}\cdot(\boldsymbol{\epsilon}\times\mathbf{k})\mathbf{A}\cdot\boldsymbol{\epsilon}_vF^0(-\frac{\mathbf{k}\cdot\mathbf{q}}{6\alpha^2},P_f\cdot k)\\
&+\{-\frac{ia}{24m_q^2}\boldsymbol{\sigma}\cdot(\boldsymbol{\epsilon}\times\mathbf{k})\boldsymbol{\epsilon}_v\cdot\mathbf{k}-\frac{ib'\omega}{12m_q\alpha^2}\boldsymbol{\sigma}\cdot(\mathbf{q}\times\boldsymbol{\epsilon}_v)\boldsymbol{\epsilon}\cdot\mathbf{q}\\
&-g_v'\frac{a\omega}{6\alpha^2}\boldsymbol{\epsilon}\cdot\mathbf{q}\mathbf{A}\cdot\boldsymbol{\epsilon}_v-g_v'\frac{a\omega}{12m_q}\boldsymbol{\epsilon}\cdot\boldsymbol{\epsilon}_v\}F^1(-\frac{\mathbf{k}\cdot\mathbf{q}}{6\alpha^2},P_f\cdot k)\\
&-g_v'\frac{a\omega}{72m_q\alpha^2}\boldsymbol{\epsilon}_v\cdot\mathbf{k}\boldsymbol{\epsilon}\cdot\mathbf{q}F^2(-\frac{\mathbf{k}\cdot\mathbf{q}}{6\alpha^2},P_f\cdot k)
\end{aligned}
\tag{43}
$$

The g-factors in Eq.(38)-(43) are defined as

$$
\langle N_f|\sum_j\hat{I}_j\boldsymbol{\sigma}_j|N_i\rangle=g_A\langle N_f|\boldsymbol{\sigma}|N_i\rangle,
\tag{44}
$$

$$
g_3^u=\langle N_f|\sum_j e_j\hat{I}_j\sigma_j^z|N_i\rangle/g_A,
\tag{45}
$$

$$
g_2^u=\langle N_f|\sum_{i\neq j} e_j\hat{I}_i\sigma_j^z|N_i\rangle/g_A,
\tag{46}
$$

$$
g_v=\langle N_f|\sum_j e_j\hat{I}_j|N_i\rangle/g_3^u g_A,
\tag{47}
$$

$$
g_v'=\frac{1}{3g_2^s g_A}\langle N_f|\sum_{i\neq j}\hat{I}_i e_j\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j|N_i\rangle,
\tag{48}
$$

$$
g_a'=\frac{1}{2g_2^s g_A}\langle N_f|\sum_{i\neq j}\hat{I}_i e_j(\boldsymbol{\sigma}_i\times\boldsymbol{\sigma}_j)_z|N_i\rangle.
\tag{49}
$$

The numerical values of these g-factors have been derived in Ref. [6] in the $SU(6)\otimes O(3)$ symmetry limit; they are listed in Table 1 for completeness.

The first terms of Eq.(42) and Eq.(43) correspond to the correlation between the magnetic transition and the c.m. motion of the meson transition operator and they contribute to the leading Born term in the u-channel. The second terms are due to correlations

between the internal and c.m. motion of the photon and meson transition operators, and they only contribute to the ground and $n \geq 1$ excited states in the harmonic oscillator basis. The last terms in both equations represent the correlation of the internal motion between the photon and meson transition operators, which only contribute to transitions between the ground and $n \geq 2$ excited states.

As pointed out before, the mass splitting between the ground state spin 1/2 and spin 3/2 is significant, the transition amplitudes for Δ resonance in ρ production or Σ^* resonance in K^* production have to be computed separately. The transition amplitude with $n = 0$ corresponding to the correlation of magnitic transitions is,

$$
\begin{aligned}
M^u(n=0) \quad &= \quad -\frac{1}{2m_q}\frac{Me^{-\frac{\mathbf{q}^2+\mathbf{k}^2}{6\alpha^2}}}{P_f\cdot k+\delta M^2/2}\frac{b'}{2m_q}\{(g_3^u g_v + g_2^u g_v')(\mathbf{k}\times\boldsymbol{\epsilon}\ )\cdot(\mathbf{q}\times\boldsymbol{\epsilon}\ _v) \\
&\quad -i(g_3^u + g_2^u g_a')\boldsymbol{\sigma}\ \cdot(\mathbf{k}\times\boldsymbol{\epsilon}\ )\times(\mathbf{q}\times\boldsymbol{\epsilon}\ _v)\}. \qquad (50)
\end{aligned}
$$

The amplitude for spin 1/2 intermediate states in the total $n = 0$ amplitudes is,

$$
\begin{aligned}
&\langle N_f|h_e|N(J=1/2)\rangle\langle N(J=1/2)|H_m|N_i\rangle \\
= \quad &\frac{\mu_N b'}{2m_q}\frac{M_f e^{-\frac{\mathbf{q}^2+\mathbf{k}^2}{6\alpha^2}}}{P_f\cdot k+\delta M^2/2}\{(\mathbf{k}\times\boldsymbol{\epsilon}\ )\cdot(\mathbf{q}\times\boldsymbol{\epsilon}\ _v) \\
&+i\boldsymbol{\sigma}\ \cdot(\mathbf{k}\times\boldsymbol{\epsilon}\ )\times(\mathbf{q}\times\boldsymbol{\epsilon}\ _v)\} \qquad (51)
\end{aligned}
$$

where μ_N is the magnetic moment, which has the following values for different processes,

$$
\mu_N = \begin{cases} \mu_\Lambda + \frac{g_{K^*\Sigma N}}{g_{K^*\Lambda N}}\mu_{\Lambda\Sigma} & \text{for} \quad \gamma N \to K^*\Lambda \\ \mu_{\Sigma^0} + \frac{g_{K^*\Lambda N}}{g_{K^*\Sigma N}}\mu_{\Lambda\Sigma} & \text{for} \quad \gamma N \to K^*\Sigma \\ \mu_{N_f} & \text{for} \quad \gamma N \to \rho N_f \end{cases} \qquad (52)
$$

Thus, we obtain the spin 3/2 resonance contribution to the transition amplitude by subtracting the spin 1/2 intermediate state contributions from the total $n = 0$ amplitudes as follows:

$$
\begin{aligned}
M^u \quad &= \quad -\frac{b'}{2m_q}\frac{M_f e^{-\frac{\mathbf{q}^2+\mathbf{k}^2}{6\alpha^2}}}{P_f\cdot k+\delta M^2/2}\{[(g_3^u g_v + g_2^u g_v')/2m_q + \mu_N](\mathbf{k}\times\boldsymbol{\epsilon}\ )\cdot(\mathbf{q}\times\boldsymbol{\epsilon}\ _v) \\
&\quad +i[(g_3^u + g_2^u g_a')/2m_q + \mu_N]\boldsymbol{\sigma}\ \cdot[(\mathbf{k}\times\boldsymbol{\epsilon}\ )\times(\mathbf{q}\times\boldsymbol{\epsilon}\ _v)]\}. \qquad (53)
\end{aligned}
$$

Substituting the g-factor coefficients into the above equation gives the following general expression for spin 3/2 resonance with $n = 0$,

$$
\begin{aligned}
M^u \quad &= \quad -\frac{b'}{2m_q}\frac{M_f g_s e^{-\frac{\mathbf{q}^2+\mathbf{k}^2}{6\alpha^2}}}{M_N(P_f\cdot k+\delta M^2/2)}\{2\boldsymbol{\sigma}\ \cdot(\mathbf{q}\times\boldsymbol{\epsilon}\ _v)\boldsymbol{\sigma}\ \cdot(\mathbf{k}\times\boldsymbol{\epsilon}\ ) \\
&\quad +i\boldsymbol{\sigma}\ \cdot[(\mathbf{q}\times\boldsymbol{\epsilon}\ _v)\times(\mathbf{k}\times\boldsymbol{\epsilon}\ )]\} \qquad (54)
\end{aligned}
$$

where the value of g_s is given in Table 1.

Note that the transition amplitudes here are generally written as operators that are similar to the CGLN amplitudes in pseudoscalar meson photoproduction. They have to be transformed into the helicity amplitudes defined in Eq.(4). In Tables 2 and 3, we show the relations between the operators presented here and the helicity amplitudes; they are generally related by the Wigner d-function.

3.3 S-channel transition amplitudes

The third term in Eq.(29) and second term in Eq.(35) are the s-channel longitudinal and transverse transition amplitudes. Following the derivation for Compton scattering in Ref. [13], we obtain the general transition amplitude for excited states in the s-channel,

$$H^J_{a\lambda_V} = \frac{2M_R}{s - M_R(M_R - i\Gamma(\mathbf{q}))} h^J_{a\lambda_V}, \tag{55}$$

where $\sqrt{s} = E_i + \omega = E_f + \omega_m$ is the total energy of the system, and $H^J_{a\lambda_V}$ are the helicity amplitudes defined above. $\Gamma(\mathbf{q})$ in Eq. 55 denotes the total width of the resonance, which is a function of the final state momentum $\mathbf{q}$. For a resonance decaying into a two-body final state with relative angular momentum l, the decay width $\Gamma(\mathbf{q})$ is given by:

$$\Gamma(\mathbf{q}) = \Gamma_R \frac{\sqrt{s}}{M_R} \sum_i x_i \left(\frac{|\mathbf{q}_i|}{|\mathbf{q}_i^R|} \right)^{2l+1} \frac{D_l(\mathbf{q}_i)}{D_l(\mathbf{q}_i^R)}, \tag{56}$$

with

$$|\mathbf{q}_i^R| = \sqrt{\frac{(M_R^2 - M_N^2 + M_i^2)^2}{4M_R^2} - M_i^2}, \tag{57}$$

and

$$|\mathbf{q}_i| = \sqrt{\frac{(s - M_N^2 + M_i^2)^2}{4s} - M_i^2}, \tag{58}$$

where x_i is the branching ratio of the resonance decaying into a meson with mass M_i and a nucleon, and Γ_R is the total decay width of the resonance with the mass M_R. The function $D_l(\mathbf{q})$ in Eq. 56, called fission barrier [17], is wavefunction dependent and has the following form in the harmonic oscillator basis:

$$D_l(\mathbf{q}) = exp\left(-\frac{\mathbf{q}^2}{3\alpha^2} \right), \tag{59}$$

which is independent of l. In principle, the branching ratio x_i should also be evaluated in the quark model.

For a given intermediate resonance state with spin J, the four independent helicity amplitudes $h^J_{a\lambda_V}$ in Eq.(55) are a combination of the meson and photon helicity amplitudes together with the Wigner-d functions

$$h^J_{a\lambda_V} = \sum_{\Lambda_f} d^J_{\Lambda_f,\Lambda_i}(\theta) A^V_{\Lambda_f} A^\gamma_{\Lambda_i}, \tag{60}$$

where $\Lambda_f = \lambda_V - \lambda_2$, $\Lambda_i = \lambda - \lambda_1$ and $\mathbf{k} \cdot \mathbf{q} = |\mathbf{k}||\mathbf{q}|cos(\theta)$.

The $A^\gamma_{1/2}$ and $A^\gamma_{3/2}$ in Eq.(60) represent the helicity amplitudes in the s-channel for the photon interactions; their explicit expressions have been given in Ref. [18].

More explicitly, the 12 independent helicity amplitudes are related to the photon helicity amplitudes $A^\gamma_{\frac{1}{2}}$, $A^\gamma_{\frac{3}{2}}$ and vector meson helicity amplitudes $S^V_{\frac{1}{2}}$, $A^V_{\frac{1}{2}}$ and $A^V_{\frac{3}{2}}$ through the following relations

$$
\begin{aligned}
h^J_{11} &= d^J_{\frac{1}{2},\frac{3}{2}}(\theta) A^V_{\frac{1}{2}} A^\gamma_{\frac{3}{2}}, \\
h^J_{10} &= d^J_{-\frac{1}{2},\frac{3}{2}}(\theta) S^V_{-\frac{1}{2}} A^\gamma_{\frac{3}{2}}, \\
h^J_{1-1} &= d^J_{-\frac{3}{2},\frac{3}{2}}(\theta) A^V_{-\frac{3}{2}} A^\gamma_{\frac{3}{2}}
\end{aligned} \tag{61}
$$

for $a = 1$, and $\lambda_V = 1, 0, -1$,

$$
\begin{aligned}
h^J_{21} &= d^J_{\frac{1}{2},\frac{1}{2}}(\theta) A^V_{\frac{1}{2}} A^\gamma_{\frac{1}{2}}, \\
h^J_{20} &= d^J_{-\frac{1}{2},\frac{1}{2}}(\theta) S^V_{-\frac{1}{2}} A^\gamma_{\frac{1}{2}}, \\
h^J_{2-1} &= d^J_{-\frac{3}{2},\frac{1}{2}}(\theta) A^V_{-\frac{3}{2}} A^\gamma_{\frac{1}{2}}
\end{aligned} \tag{62}
$$

for $a = 2$, and $\lambda_V = 1, 0, -1$,

$$
\begin{aligned}
h^J_{31} &= d^J_{\frac{3}{2},\frac{3}{2}}(\theta) A^V_{\frac{3}{2}} A^\gamma_{\frac{3}{2}}, \\
h^J_{30} &= d^J_{\frac{1}{2},\frac{3}{2}}(\theta) S^V_{\frac{1}{2}} A^\gamma_{\frac{3}{2}}, \\
h^J_{3-1} &= d^J_{-\frac{1}{2},\frac{3}{2}}(\theta) A^V_{-\frac{1}{2}} A^\gamma_{\frac{3}{2}}
\end{aligned} \tag{63}
$$

for $a = 3$, and $\lambda_V = 1, 0, -1$, and

$$
\begin{aligned}
h^J_{41} &= d^J_{\frac{3}{2},\frac{1}{2}}(\theta) A^V_{\frac{3}{2}} A^\gamma_{\frac{1}{2}}, \\
h^J_{40} &= d^J_{\frac{1}{2},\frac{1}{2}}(\theta) S^V_{\frac{1}{2}} A^\gamma_{\frac{1}{2}}, \\
h^J_{4-1} &= d^J_{-\frac{1}{2},\frac{1}{2}}(\theta) A^V_{-\frac{1}{2}} A^\gamma_{\frac{1}{2}}
\end{aligned} \tag{64}
$$

for $a = 4$, and $\lambda_V = 1, 0, -1$.

The amplitudes with negative helicities in the above equations are not independent from those with positive one; they are related by an additional phase factor according to the Wigner-Eckart theorem,

$$
A^V_{-\lambda} = (-1)^{J_f - J_i - J_V} A^V_\lambda \tag{65}
$$

where J_f and J_i are the final nucleon and initial resonance spins, and J_V is the angular momentum of the vector meson. The angular distributions of the helicity amplitudes in terms of the multipole transitions have been discussed in Ref. [19], the expressions here are consistent with their analysis.

The evaluation of the vector meson helicity amplitudes are similar to that of the photon amplitudes. The transition operator for a resonance decaying into a vector meson and a nucleon is,

$$H_m^T=\sum_l\{i\frac{b'}{2m_q}\boldsymbol{\sigma}_{\ l}\cdot(\mathbf{q}\times\boldsymbol{\epsilon}_{\ v})+\frac{a}{2\mu_q}\mathbf{p}_l'\cdot\boldsymbol{\epsilon}_{\ v}\}\hat{I}_l e^{-i\mathbf{q}\cdot\mathbf{r}_l}, \tag{66}$$

for transverse transitions and

$$H_m^L=\frac{a\mu}{|\mathbf{q}|}\sum_l\hat{I}_l e^{-i\mathbf{q}\cdot\mathbf{r}_l} \tag{67}$$

for longitudinal transitions. Thus, H_m^T and H_m^L have the group structure,

$$H_m^T=\hat{I}_3(AL_{(3)}^-+B\sigma_{(3)}^-), \tag{68}$$

and

$$H_m^L=\hat{I}_3 S, \tag{69}$$

where

$$A=\frac{3a}{2\sqrt{2}m_q}\langle\psi_f|p_3^- e^{-i\mathbf{q}\cdot\mathbf{r}_3}|\psi_R\rangle, \tag{70}$$

$$B=\frac{-3b'}{2m_q}|\mathbf{q}|\langle\psi_f|e^{-i\mathbf{q}\cdot\mathbf{r}_3}|\psi_R\rangle, \tag{71}$$

$$S=-\frac{3\mu a}{|\mathbf{q}|}\langle\psi_f|e^{-i\mathbf{q}\cdot\mathbf{r}_3}|\psi_R\rangle. \tag{72}$$

where $p_3^-=p_x-ip_y$. In Eq.(68), $L_{(3)}^-$ and $\sigma_{(3)}^-$ denote orbital and spin flip operators. The helicity amplitudes $A_{\frac{1}{2}}^V$, $A_{\frac{3}{2}}^V$ and $S_{\frac{1}{2}}^V$ are the matrix elements of Eq.(68) and Eq.(69). We list the angular momentum and flavor parts of $A_{\frac{1}{2}}^V$, $A_{\frac{3}{2}}^V$ and $S_{\frac{1}{2}}^V$ for ω and ρ photoproduction in Tables 4-6 in the $SU(6)\otimes O(3)$ limit with A, B and S in the second row to denote the corresponding spatial integrals, which are given in Table 7.

The resonances with $n\geq 3$ are treated as degenerate since there is little information available about them. Their longitudinal transition in the s-channel is given by:

$$h_{a\lambda_V=0}^J=(M_3^s(L)+M_2^s(L))e^{-\frac{\mathbf{q}^2+\mathbf{k}^2}{6\alpha^2}} \tag{73}$$

where

$$\begin{aligned}M_3^s(L) &= g_3^s\frac{a\mu}{|\mathbf{q}|}\{-\frac{i}{2m_q}\boldsymbol{\sigma}\ \cdot(\boldsymbol{\epsilon}\ \times\mathbf{k})\frac{1}{n!}(\frac{\mathbf{k}\cdot\mathbf{q}}{3\alpha^2})^n\\ &\quad +g_v\frac{\omega}{3\alpha^2}\mathbf{q}\cdot\boldsymbol{\epsilon}\ \frac{1}{(n-1)!}(\frac{\mathbf{k}\cdot\mathbf{q}}{3\alpha^2})^{n-1}\},\end{aligned} \tag{74}$$

and

$$\begin{aligned}M_2^s(L) &= -g_2^u\frac{a\mu}{|\mathbf{q}|}\{g_a'\frac{i}{2m_q}\boldsymbol{\sigma}\ \cdot(\boldsymbol{\epsilon}\ \times\mathbf{k})\frac{1}{n!}(\frac{-\mathbf{k}\cdot\mathbf{q}}{6\alpha^2})^n\\ &\quad +g_v'\frac{\omega}{6\alpha^2}\mathbf{q}\cdot\boldsymbol{\epsilon}\ \frac{1}{(n-1)!}(\frac{-\mathbf{k}\cdot\mathbf{q}}{6\alpha^2})^{n-1}\}.\end{aligned} \tag{75}$$

The g-factors in Eq.(74) and (75) have been defined previously, and

$$g_3^s = \langle N_f | \sum_j \hat{I}_j e_j \sigma_j^z | N_i \rangle / g_A = e_m + g_3^u, \tag{76}$$

where e_m is the charge of the outgoing vector meson.

The transverse transition amplitudes at the quark level are:

$$h^J_{a\lambda_V=\pm 1} = (M_3^s(T) + M_2^s(T)) e^{-\frac{\mathbf{q}^2+\mathbf{k}^2}{6\alpha^2}} \tag{77}$$

where

$$\begin{aligned} M_3^s(T)/g_3^s &= \frac{b'}{4m_q^2}\{g_v(\mathbf{q}\times\boldsymbol{\epsilon}_v)\cdot(\boldsymbol{\epsilon}\times\mathbf{k}) + i\boldsymbol{\sigma}\cdot(\mathbf{q}\times\boldsymbol{\epsilon}_v)\times(\boldsymbol{\epsilon}\times\mathbf{k})\}\frac{1}{n!}(\frac{\mathbf{k}\cdot\mathbf{q}}{3\alpha^2})^n \\ &+\{-\frac{ia}{12m_q^2}\boldsymbol{\sigma}\cdot(\boldsymbol{\epsilon}\times\mathbf{k})\boldsymbol{\epsilon}_v\cdot\mathbf{k} + \frac{ib'\omega}{6m_q\alpha^2}\boldsymbol{\sigma}\cdot(\mathbf{q}\times\boldsymbol{\epsilon}_v)\boldsymbol{\epsilon}\cdot\mathbf{q} \\ &+g_v\frac{a\omega}{6m_q}\boldsymbol{\epsilon}_v\cdot\boldsymbol{\epsilon}\}\frac{1}{(n-1)!}(\frac{\mathbf{k}\cdot\mathbf{q}}{3\alpha^2})^{n-1} \\ &+g_v\frac{a\omega}{18m_q\alpha^2}\boldsymbol{\epsilon}_v\cdot\mathbf{k}\boldsymbol{\epsilon}\cdot\mathbf{q}\frac{1}{(n-2)!}(\frac{\mathbf{k}\cdot\mathbf{q}}{3\alpha^2})^{n-2}, \end{aligned} \tag{78}$$

and

$$\begin{aligned} M_2^s(T)/g_2^u &= \frac{b'}{4m_q^2}\{g_v'(\mathbf{q}\times\boldsymbol{\epsilon}_v)\cdot(\boldsymbol{\epsilon}\times\mathbf{k}) + ig_a'\boldsymbol{\sigma}\cdot(\mathbf{q}\times\boldsymbol{\epsilon}_v)\times(\boldsymbol{\epsilon}\times\mathbf{k})\}\frac{1}{n!}(\frac{-\mathbf{k}\cdot\mathbf{q}}{6\alpha^2})^n \\ &+\{\frac{ia}{24m_q^2}\boldsymbol{\sigma}\cdot(\boldsymbol{\epsilon}\times\mathbf{k})\boldsymbol{\epsilon}_v\cdot\mathbf{k} - \frac{ib'\omega}{12m_q\alpha^2}\boldsymbol{\sigma}\cdot(\mathbf{q}\times\boldsymbol{\epsilon}_v)\boldsymbol{\epsilon}\cdot\mathbf{q} \\ &-g_v'\frac{a\omega}{12m_q}\boldsymbol{\epsilon}_v\cdot\boldsymbol{\epsilon}\}\frac{1}{(n-1)!}(\frac{-\mathbf{k}\cdot\mathbf{q}}{6\alpha^2})^{n-1} \\ &+g_v'\frac{a\omega}{72m_q\alpha^2}\boldsymbol{\epsilon}_v\cdot\mathbf{k}\boldsymbol{\epsilon}\cdot\mathbf{q}\frac{1}{(n-2)!}(\frac{-\mathbf{k}\cdot\mathbf{q}}{6\alpha^2})^{n-2}. \end{aligned} \tag{79}$$

Qualitatively, we find that the resonances with larger partial waves have larger decay widths into the vector meson and nucleon though this is not as explicit as in the pseudoscalar case [6, 20]. Thus, we could use the mass and decay width of the high spin states, such as $G_{17}(2190)$ for $n = 3$ states and $H_{19}(2220)$ for $n = 4$ states in the ω photoproduction. The relation between these operators and the helicity amplitudes $h_{a\lambda_V}$ has been given in Table 2 and 3.

4. Discussion and conclusion

In this paper we have developed the framework and formalism for the description on vector meson photoproduction in the constituent quark model. The use of an effective Lagrangian allows gauge invariance to be satisfied straightforwardly. The advantage of

using the quark model approach is that the number of free parameters is greatly reduced in comparison to hadronic models that introduce each resonance as a new independent field with unknown coupling constants. In our approach, there are only three parameters in the $SU(6)\otimes O(3)$ symmetry limit, the coupling constants a and b which determine the coupling strengths of the vector meson to the quark, and the harmonic oscillator strength α^2.

One significant approximation inherent in the presented approach is the treatment of the vector mesons as point particles, thus, the effects due to the finite size of the vector mesons that were important in the $^3P_0$ model are neglected here. A possible way that may partly compensate this problem is adjusting the parameter α^2, the harmonic oscillator strength. In general, the question of how to include the finite size of vector mesons while maintaining gauge invariance is very complicated and has not yet been resolved.

While this approach should give a qualitative description of vector meson photoproduction, a precise quantitative agreement with the data cannot be expected, as configuration mixing effects for the resonances in the second and third resonance region are known to be very important. Such effects could be investigated in our approach by inserting a mixing parameter C_R in front of the transition amplitudes for the s-channel resonances, as was done in Ref. [20]. Yet even without new parameters the present approach should be able to answer basic questions such as the need for an additional pomeron or other t-channel exchanges in light of the duality argument. We believe that the model presented here provides a systematic method to investigate the resonance behavior in the vector meson production for the first time. It therefore can assist in answering lingering quetions about the existence of the "missing resonances". The numerical implementation of this approach for ρ and ω photoproduction is in progress and will be published elsewhere.

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Appendix

The matrix element for the nucleon pole term of transverse excitations in the s-channel is,

$$\begin{aligned} M_N^s(T) &= -\frac{M_N e^{-\frac{\mathbf{q}^2+\mathbf{k}^2}{6\alpha^2}}}{P_N\cdot k}\{g_v^t\frac{\omega a e_N}{E_f+M_f}\boldsymbol{\epsilon}_v\cdot\boldsymbol{\epsilon}-g_A\mu_N\frac{b'}{2m_q}[(\boldsymbol{\epsilon}_v\times\mathbf{q})\cdot(\boldsymbol{\epsilon}\times\mathbf{k}) \\ &\quad +i\boldsymbol{\sigma}\cdot(\boldsymbol{\epsilon}_v\times\mathbf{q})\times(\boldsymbol{\epsilon}\times\mathbf{k})]\}, \end{aligned} \tag{80}$$

while the one for the u-channel is,

$$\begin{aligned} M_N^u(T) &= -\frac{M_f e^{-\frac{\mathbf{q}^2+\mathbf{k}^2}{6\alpha^2}}}{P_f\cdot k}\{g_v^t\frac{\omega a e_f}{E_N+M_N}\boldsymbol{\epsilon}\cdot\boldsymbol{\epsilon}_v+g_A\mu_f\frac{b'}{2m_q}[(\boldsymbol{\epsilon}\times\mathbf{k})\cdot(\boldsymbol{\epsilon}_v\times\mathbf{q}) \\ &\quad +i\boldsymbol{\sigma}\cdot((\boldsymbol{\epsilon}\times\mathbf{k})\times(\boldsymbol{\epsilon}_v\times\mathbf{q}))]\} \\ &\quad +\frac{e_f e^{-\frac{\mathbf{q}^2+\mathbf{k}^2}{6\alpha^2}}}{P_f\cdot k}\{\frac{-g_v^t a}{E_N+M_N}\mathbf{q}\cdot\boldsymbol{\epsilon}\,\mathbf{k}\cdot\boldsymbol{\epsilon}_v+ig_A\frac{b'}{2m_q}\boldsymbol{\sigma}\cdot(\boldsymbol{\epsilon}_v\times\mathbf{q})\mathbf{q}\cdot\boldsymbol{\epsilon}\}. \end{aligned} \tag{81}$$

The matrix element for the nucleon pole term of the longitudinal excitations in the s-channel is,

$$M_N^s(L) = -g_v^t \frac{i\mu a}{|\mathbf{q}|} \frac{M_N}{P_N \cdot k} \mu_N \boldsymbol{\sigma} \cdot (\boldsymbol{\epsilon} \times \mathbf{k}) e^{-\frac{\mathbf{q}^2+\mathbf{k}^2}{6\alpha^2}}, \tag{82}$$

while the one for the u-channel is,

$$M_N^u(L) = g_v^t \frac{\mu a}{|\mathbf{q}|} \frac{1}{P_f \cdot k} \{-e_f \mathbf{q} \cdot \boldsymbol{\epsilon} + iM_f \mu_f \boldsymbol{\sigma} \cdot (\boldsymbol{\epsilon} \times \mathbf{k})\} e^{-\frac{\mathbf{q}^2+\mathbf{k}^2}{6\alpha^2}}, \tag{83}$$

where the g-factor g_v^t has the following form,

$$g_v^t = \langle N_f | \sum_j \hat{I}_j | N_i \rangle. \tag{84}$$

The t-channel matrix element for the transverse transition is,

$$\begin{aligned} M^t(T) \quad = \quad & \frac{ae_m}{q \cdot k} \{-g_v^t [\omega_m + (\frac{\mathbf{q}}{E_f + M_f} + \frac{\mathbf{k}}{E_N + M_N}) \cdot \mathbf{q}] \boldsymbol{\epsilon} \cdot \boldsymbol{\epsilon}_v \\ & + g_A \frac{i}{2m_q} \boldsymbol{\sigma} \cdot (\mathbf{k} \times \mathbf{q}) \boldsymbol{\epsilon} \cdot \boldsymbol{\epsilon}_v \\ & - g_v^t (\frac{1}{E_f + M_f} + \frac{1}{E_N + M_N}) \mathbf{q} \cdot \boldsymbol{\epsilon}\, \mathbf{k} \cdot \boldsymbol{\epsilon}_v \\ & + g_A \frac{i}{2m_q} \boldsymbol{\sigma} \cdot ((\mathbf{k} - \mathbf{q}) \times \boldsymbol{\epsilon}_v) \mathbf{q} \cdot \boldsymbol{\epsilon} \\ & + g_A \frac{i}{2m_q} \boldsymbol{\sigma} \cdot ((\mathbf{k} - \mathbf{q}) \times \boldsymbol{\epsilon}) \mathbf{k} \cdot \boldsymbol{\epsilon}_v \} e^{-\frac{(\mathbf{k}-\mathbf{q})^2}{6\alpha^2}}, \end{aligned} \tag{85}$$

and for the longitudinal transition is,

$$M^t(L) = \frac{\mu}{|\mathbf{q}|} \frac{ae_m}{q \cdot k} \{g_v^t (1 - \frac{\omega}{E_f + M_f}) \mathbf{q} \cdot \boldsymbol{\epsilon} + g_A \frac{i\omega}{2m_q} \boldsymbol{\sigma} \cdot ((\mathbf{k} - \mathbf{q}) \times \boldsymbol{\epsilon})\} e^{-\frac{(\mathbf{k}-\mathbf{q})^2}{6\alpha^2}}. \tag{86}$$

References

[1] F.J. Klein, Ph.D. thesis, University of Bonn (1996), and to be published.

[2] TJNAF experimental proposals E94-109 (P. Cole, R. Whitney, Co-spokesmen); E91-024 (V. Burkert, H. Funsten. M. Manley, B. Mecking, Co-spokesmen).

[3] N. Isgur and G. Karl, Phys. Letts. **72B**, 109(1977); Phys. Rev. **D23**, 817(1981); R. Koniuk and N. Isgur, Phys. Rev. **D21**, 1868(1980).

[4] S. Capstick, and W. Roberts, Phys. Rev. **D49**, 4570(1994).

[5] P. Söding, Phys. Lett. **19**, 702(1965).

[6] Zhenping Li,*et al*, Phys. Rev. **C56**, 1099 (1997).

[7] Alain L.E. Yaouanc *et al.* “Hadron Transitions In The Quark Model“ Gordon and Breach Science Publishers, New York, 1988.

[8] M. Pichowsky, Ç. Şavklı, F. Tabakin, Phys. Rev. **C53**, 593 (1996).

[9] C. G. Fasano, F. Tabakin and B. Saghai, Phys. Rev. **C46**, 2430(1992).

[10] K. Schilling, P. Seyboth, and G. Wolf, Nucl. Phys. **B15**, 397(1970); **B18**, 332(E) (1970).

[11] J.Ballam *et al*, Phys. Rev. **D5**, 545 (1972).

[12] Bjorken and Drell, “Relativistic Quantum Mechanics“

[13] Zhenping Li, Phys. Rev. **D48**, 3070(1993).

[14] R. Dolen, D. Horn, and C. Schmid, Phys. Rev. **166**, 1768(1966); R. Williams, C. R.Ji, and S. Cotanch, Phys. Rev. **C46**, 1617(1992), *ibid*, **C43**, 452(1991).

[15] M. Pichowsky, “Nonperturbative Quark Dynamics in diffractive processe“, Ph.D. Dissertation, The University Of Pittsburgh, (1996); and references therein.

[16] S. Capstick and B. D. Keister, Phys. Rev. **D46**, 84 (1992).

[17] J. M. Blatt and V. F. Weisskopf, Theoretical Nuclear Physics, (Wiley, New York, 1952), p361.

[18] Zhenping Li, “Photo- and Electroproduction of Baryon Resonances in A Potential Quark Mode“, Ph.D. Dissertation, The University of Tennessee, (1991).

[19] Ç. Şavklı, F. Tabakin and S.N. Yang, Phys. Rev. **C53**, 1132(1996).

[20] Zhenping Li, Phys. Rev. **D52**, 4961 (1995).

Table 1: The g-factors in the u-channel amplitudes in Eqs. 42 and 43 for different production processes.

Reactions	g_3^u	g_2^u	g_v	g_v'	g_a'	g_A	g_S	g_v^t
$\gamma p \to K^{*+}\Lambda$	-$\frac{1}{3}$	$\frac{1}{3}$	1	1	1	$\sqrt{\frac{3}{2}}$	-$\frac{\mu_\Lambda}{3}$	$\sqrt{\frac{3}{2}}$
$\gamma n \to K^{*0}\Lambda$	-$\frac{1}{3}$	$\frac{1}{3}$	1	-1	-1	$\sqrt{\frac{3}{2}}$	$\frac{\mu_\Lambda}{3}$	$\sqrt{\frac{3}{2}}$
$\gamma p \to K^{*+}\Sigma^0$	-$\frac{1}{3}$	$\frac{1}{3}$	-3	-7	9	-$\frac{1}{3\sqrt{2}}$	μ_{Σ^0}	$\frac{1}{\sqrt{2}}$
$\gamma n \to K^{*0}\Sigma^0$	-$\frac{1}{3}$	$\frac{1}{3}$	-3	11	-9	$\frac{1}{3\sqrt{2}}$	μ_{Σ^0}	$-\frac{1}{\sqrt{2}}$
$\gamma p \to K^{*0}\Sigma^+$	-$\frac{1}{3}$	$\frac{4}{3}$	-3	2	0	$\frac{1}{3}$	$\frac{2\mu_{\Sigma^+}}{3}$	0
$\gamma n \to K^{*+}\Sigma^-$	-$\frac{1}{3}$	-$\frac{2}{3}$	-3	2	0	-$\frac{1}{3}$	0	1
$\gamma p \to \omega p$	1	0	1	0	0	1	0	3
$\gamma n \to \omega n$	-$\frac{2}{3}$	$\frac{2}{3}$	0	-1	0	1	0	3
$\gamma p \to \rho^+ n$	-$\frac{1}{3}$	$\frac{1}{3}$	$\frac{3}{5}$	$\frac{1}{5}$	$\frac{9}{5}$	$\frac{5}{3}$	-$\frac{2\mu_n}{5}$	1
$\gamma n \to \rho^- p$	$\frac{2}{3}$	$\frac{1}{3}$	$\frac{3}{5}$	$\frac{1}{5}$	- $\frac{9}{5}$	- $\frac{5}{3}$	-$\frac{4\mu_p}{15}$	1
$\gamma p \to \rho^0 p$	$\frac{7}{15}$	$\frac{8}{15}$	$\frac{15}{7}$	2	0	$\frac{5}{3\sqrt{2}}$	$\frac{8\mu_p}{15}$	$-\frac{1}{\sqrt{2}}$
$\gamma n \to \rho^0 n$	-$\frac{2}{15}$	$\frac{2}{15}$	6	-7	0	-$\frac{5}{3\sqrt{2}}$	$\frac{4\mu_n}{5}$	$\frac{1}{\sqrt{2}}$

Table 2: The operators in the longitudinal excitations expressed in terms of the helicity amplitudes. $\hat{\mathbf{k}}$ and $\hat{\mathbf{q}}$ are the unit vectors of $\mathbf{k}$ and $\mathbf{q}$, respectively. Other components of $H_{a\lambda_V}$ are zero. The d functions have the rotation angle θ.

Operators	$H_{10}(\lambda_2 = 1/2)$ $H_{30}(\lambda_2 = -1/2)$	$H_{20}(\lambda_2 = 1/2)$ $H_{40}(\lambda_2 = -1/2)$
$\hat{\mathbf{q}} \cdot \boldsymbol{\epsilon}$	$d^1_{10} d^{\frac{1}{2}}_{-\frac{1}{2}\lambda_2}$	$d^1_{10} d^{\frac{1}{2}}_{\frac{1}{2}\lambda_2}$
$\boldsymbol{\sigma} \cdot (\boldsymbol{\epsilon} \times \hat{\mathbf{k}})$	$i\sqrt{2} d^{\frac{1}{2}}_{\frac{1}{2}\lambda_2}$	0
$\boldsymbol{\sigma} \cdot (\boldsymbol{\epsilon} \times \hat{\mathbf{q}})$	$i d^1_{10} d^{\frac{1}{2}}_{-\frac{1}{2}\lambda_2}$ $+ i\sqrt{2} d^1_{00} d^{\frac{1}{2}}_{\frac{1}{2}\lambda_2}$	$-i d^1_{10} d^{\frac{1}{2}}_{\frac{1}{2}\lambda_2}$

Table 3: The operators in the transverse excitations expressed in terms of the helicity amplitudes. $\hat{\mathbf{k}}$ and $\hat{\mathbf{q}}$ are the unit vectors of $\mathbf{k}$ and $\mathbf{q}$, respectively. The d functions have the rotation angle θ. $\lambda_V = \pm 1$ denote the helicities of the mesons, and the vector $\hat{\mathbf{Z}}$ is defined as $\hat{\mathbf{Z}} = (\boldsymbol{\epsilon} \ \times \hat{\mathbf{k}}) \times (\boldsymbol{\epsilon}_{\ v} \times \hat{\mathbf{q}})$.

Operators	$H_{1\lambda_V}(\lambda_2 = 1/2)$ $H_{3\lambda_V}(\lambda_2 = -1/2)$	$H_{2\lambda_V}(\lambda_2 = 1/2)$ $H_{4\lambda_V}(\lambda_2 = -1/2)$
$(\boldsymbol{\epsilon} \ \times \hat{\mathbf{k}}) \cdot (\boldsymbol{\epsilon}_{\ v} \times \hat{\mathbf{q}})$	$-\lambda_V d^1_{1\lambda_V} d^{\frac{1}{2}}_{-\frac{1}{2}\lambda_2}$	$-\lambda_V d^1_{1\lambda_V} d^{\frac{1}{2}}_{\frac{1}{2}\lambda_2}$
$\boldsymbol{\sigma} \ \cdot \hat{\mathbf{Z}}$	$-i\lambda_V d^1_{11} d^{\frac{1}{2}}_{-\frac{1}{2}\lambda_2}$	$i\lambda_V d^1_{11} d^{\frac{1}{2}}_{\frac{1}{2}\lambda_2}$
	$-i\sqrt{2}\lambda_V d^1_{0\lambda_V} d^{\frac{1}{2}}_{\frac{1}{2}\lambda_2}$	
$\boldsymbol{\sigma} \ \cdot (\boldsymbol{\epsilon} \ \times \hat{\mathbf{k}})\hat{\mathbf{k}} \cdot \boldsymbol{\epsilon}_{\ v}$	$i\sqrt{2} d^1_{0\lambda_V} d^{\frac{1}{2}}_{\frac{1}{2}\lambda_2}$	0
$\boldsymbol{\sigma} \ \cdot (\boldsymbol{\epsilon}_{\ v} \times \hat{\mathbf{q}})\hat{\mathbf{q}} \cdot \boldsymbol{\epsilon}$	$i\sqrt{2}\lambda_V d^1_{10} d^1_{1\lambda_V} d^{\frac{1}{2}}_{\frac{1}{2}\lambda_2}$	$i\sqrt{2}\lambda_V d^1_{10} d^1_{-1\lambda_V} d^{\frac{1}{2}}_{-\frac{1}{2}\lambda_2}$
	$-i\lambda_V d^1_{10} d^1_{0\lambda_V} d^{\frac{1}{2}}_{-\frac{1}{2}\lambda_2}$	$+i\lambda_V d^1_{10} d^1_{0\lambda_V} d^{\frac{1}{2}}_{\frac{1}{2}\lambda_2}$
$\boldsymbol{\epsilon}_{\ v} \cdot \boldsymbol{\epsilon}$	$d^1_{1\lambda_V} d^{\frac{1}{2}}_{-\frac{1}{2}\lambda_2}$	$d^1_{1\lambda_V} d^{\frac{1}{2}}_{\frac{1}{2}\lambda_2}$
$\hat{\mathbf{q}} \cdot \boldsymbol{\epsilon} \ \hat{\mathbf{k}} \cdot \boldsymbol{\epsilon}_{\ v}$	$d^1_{0\lambda_V} d^1_{01} d^{\frac{1}{2}}_{-\frac{1}{2}\lambda_2}$	$d^1_{0\lambda_V} d^1_{01} d^{\frac{1}{2}}_{\frac{1}{2}\lambda_2}$
$\boldsymbol{\sigma} \ \cdot (\boldsymbol{\epsilon} \ \times \hat{\mathbf{q}})\hat{\mathbf{k}} \cdot \boldsymbol{\epsilon}_{\ v}$	$i\sqrt{2} d^1_{00} d^1_{0\lambda_V} d^{\frac{1}{2}}_{\frac{1}{2}\lambda_2}$	
	$+i d^1_{10} d^1_{0\lambda_V} d^{\frac{1}{2}}_{\frac{1}{2}\lambda_2}$	$-i d^1_{10} d^1_{0\lambda_V} d^{\frac{1}{2}}_{\frac{1}{2}\lambda_2}$
$\boldsymbol{\sigma} \ \cdot (\boldsymbol{\epsilon}_{\ v} \times \hat{\mathbf{k}})\hat{\mathbf{q}} \cdot \boldsymbol{\epsilon}$	$-i\sqrt{2} d^1_{10} d^1_{-1\lambda_V} d^{\frac{1}{2}}_{\frac{1}{2}\lambda_2}$	$-i\sqrt{2} d^1_{10} d^1_{1\lambda_V} d^{\frac{1}{2}}_{-\frac{1}{2}\lambda_2}$
$\boldsymbol{\sigma} \ \cdot (\boldsymbol{\epsilon}_{\ v} \times \boldsymbol{\epsilon} \ )$	$-i\sqrt{2} d^1_{0\lambda_V} d^{\frac{1}{2}}_{\frac{1}{2}\lambda_2}$	
	$-i d^1_{1\lambda_V} d^{\frac{1}{2}}_{-\frac{1}{2}\lambda_2}$	$i d^1_{1\lambda_V} d^{\frac{1}{2}}_{\frac{1}{2}\lambda_2}$
$\boldsymbol{\sigma} \ \cdot (\hat{\mathbf{k}} \times \hat{\mathbf{q}})\boldsymbol{\epsilon} \ \ \cdot \boldsymbol{\epsilon}_{\ v}$	$i\sqrt{2} d^1_{-10} d^1_{1\lambda_V} d^{\frac{1}{2}}_{\frac{1}{2}\lambda_2}$	$i\sqrt{2} d^1_{10} d^1_{1\lambda_V} d^{\frac{1}{2}}_{-\frac{1}{2}\lambda_2}$

Table 4: The agular momentum and flavor parts of the helicity amplitudes for $\gamma p \to \omega p$, or $\gamma n \to \omega n$, in the $SU(6)\otimes O(3)$ symmetry limit. They are the coefficients of the spatial integrals A, B and S in Eq.(68) and (69). The analytic expressions for A, B and S in Table 4-6 are given in Table 7. The "*" in Table 4-6 denotes those states decoupling in spin and flavor space, thus, their amplitudes are zero.

States	$S_{1/2}$	$A_{1/2}$		$A_{3/2}$	
	S	A	B	A	B
$N(^2P_M)_{\frac{1}{2}^-}$	0	0	$-\frac{2}{3\sqrt{6}}$	0	0
$N(^2P_M)_{\frac{3}{2}^-}$	0	0	$\frac{2}{3\sqrt{3}}$	0	0
$N(^4P_M)_{\frac{1}{2}^-}$	0	0	$-\frac{1}{3\sqrt{6}}$	*	*
$N(^4P_M)_{\frac{3}{2}^-}$	0	0	$\frac{1}{3\sqrt{30}}$	0	$-\frac{1}{\sqrt{15}}$
$N(^4P_M)_{\frac{5}{2}^-}$	0	0	$\frac{1}{\sqrt{30}}$	0	$\frac{1}{\sqrt{10}}$
$N(^2D_S)_{\frac{3}{2}^+}$	$-\sqrt{\frac{2}{5}}$	$\sqrt{\frac{3}{5}}$	$-\frac{1}{3}\sqrt{\frac{2}{5}}$	$-\frac{1}{\sqrt{5}}$	*
$N(^2D_S)_{\frac{5}{2}^+}$	$\sqrt{\frac{3}{5}}$	$\sqrt{\frac{2}{5}}$	$\frac{1}{3}\sqrt{\frac{3}{5}}$	$\frac{2}{\sqrt{5}}$	*
$N(^2S'_S)_{\frac{1}{2}^+}$	1	*	$\frac{1}{3}$	*	*
$N(^2S_M)_{\frac{1}{2}^+}$	0	*	$\frac{2}{3\sqrt{2}}$	*	*
$N(^4S_M)_{\frac{3}{2}^+}$	0	*	$\frac{1}{3\sqrt{2}}$	*	$\frac{1}{\sqrt{6}}$
$N(^2D_M)_{\frac{3}{2}^+}$	0	0	$-\frac{2}{3\sqrt{5}}$	0	*
$N(^2D_M)_{\frac{5}{2}^+}$	0	0	$\sqrt{\frac{2}{15}}$	0	*
$N(^4D_M)_{\frac{1}{2}^+}$	0	0	$\frac{1}{3\sqrt{10}}$	*	*
$N(^4D_M)_{\frac{3}{2}^+}$	0	0	$-\frac{1}{3\sqrt{10}}$	0	$\frac{1}{\sqrt{30}}$
$N(^4D_M)_{\frac{5}{2}^+}$	0	0	$-\frac{1}{\sqrt{210}}$	0	$-\sqrt{\frac{3}{35}}$
$N(^4D_M)_{\frac{7}{2}^+}$	0	0	$\frac{1}{\sqrt{35}}$	0	$\frac{1}{\sqrt{21}}$

Table 5: The angular momentum and flavor parts of the helicity amplitudes for $\gamma p \to \rho^0 p$ in the $SU(6) \otimes O(3)$ symmetry limit, while those for $\gamma n \to \rho^0 n$ are given by $A(\gamma n \to \rho^0 n) = (-1)^{I+1/2} A(\gamma p \to \rho^0 p)$, where I is the isospin of the resonances.

States	$S_{1/2}$	$A_{1/2}$		$A_{3/2}$	
	S	A	B	A	B
$N(^2P_M)_{\frac{1}{2}^-}$	$\frac{1}{3\sqrt{3}}$	$-\frac{\sqrt{2}}{3\sqrt{3}}$	$\frac{2}{9\sqrt{3}}$	*	*
$N(^2P_M)_{\frac{3}{2}^-}$	$-\frac{1}{3}\sqrt{\frac{2}{3}}$	$-\frac{1}{3\sqrt{3}}$	$-\frac{2}{9}\sqrt{\frac{2}{3}}$	$-\frac{1}{3}$	*
$N(^4P_M)_{\frac{1}{2}^-}$	0	0	$-\frac{1}{18\sqrt{3}}$	*	*
$N(^4P_M)_{\frac{3}{2}^-}$	0	0	$\frac{1}{18\sqrt{15}}$	0	$\frac{1}{6\sqrt{5}}$
$N(^4P_M)_{\frac{5}{2}^-}$	0	0	$\frac{1}{6\sqrt{15}}$	0	$\frac{1}{\sqrt{30}}$
$\Delta(^2P_M)_{\frac{1}{2}^-}$	$-\frac{1}{3\sqrt{3}}$	$\frac{1}{3}\sqrt{\frac{2}{3}}$	$\frac{1}{9\sqrt{3}}$	*	*
$\Delta(^2P_M)_{\frac{3}{2}^-}$	$\frac{1}{3}\sqrt{\frac{2}{3}}$	$\frac{1}{3\sqrt{3}}$	$-\frac{1}{9}\sqrt{\frac{2}{3}}$	$\frac{1}{3}$	*
$N(^2D_S)_{\frac{3}{2}^+}$	$\frac{1}{3\sqrt{5}}$	$-\frac{1}{\sqrt{30}}$	$\frac{1}{9\sqrt{5}}$	$\frac{1}{3\sqrt{10}}$	*
$N(^2D_S)_{\frac{5}{2}^+}$	$-\frac{1}{\sqrt{30}}$	$-\frac{1}{3\sqrt{5}}$	$-\frac{5}{3\sqrt{30}}$	$-\frac{2}{3\sqrt{10}}$	*
$\Delta(^4D_S)_{\frac{1}{2}^+}$	0	0	$\frac{2}{9\sqrt{5}}$	*	*
$\Delta(^4D_S)_{\frac{3}{2}^+}$	0	0	$-\frac{2}{9\sqrt{5}}$	0	$\frac{2}{3\sqrt{15}}$
$\Delta(^4D_S)_{\frac{5}{2}^+}$	0	0	$-\frac{2}{9}\sqrt{\frac{3}{35}}$	0	$-\frac{2}{3}\sqrt{\frac{6}{35}}$
$\Delta(^4D_S)_{\frac{7}{2}^+}$	0	0	$\frac{2}{3}\sqrt{\frac{2}{35}}$	0	$\frac{4}{3\sqrt{42}}$
$N(^2S'_S)_{\frac{1}{2}^+}$	$-\frac{1}{3\sqrt{2}}$	*	$-\frac{5}{9\sqrt{2}}$	*	*
$\Delta(^4S'_S)_{\frac{3}{2}^+}$	0	*	$\frac{2}{9}$	*	$\frac{2}{3\sqrt{3}}$
$\Delta(^4S_S)_{\frac{3}{2}^+}$	0	*	$\frac{2}{9}$	*	$\frac{2}{3\sqrt{3}}$
$N(^2S_M)_{\frac{1}{2}^+}$	$-\frac{1}{3}$	*	$-\frac{2}{9}$	*	*
$N(^4S_M)_{\frac{3}{2}^+}$	0	*	$\frac{1}{18}$	*	$\frac{1}{6\sqrt{3}}$
$\Delta(^2S_M)_{\frac{1}{2}^+}$	$\frac{1}{3}$	*	$-\frac{1}{9}$	*	*
$N(^2D_M)_{\frac{3}{2}^+}$	$\frac{1}{3}\sqrt{\frac{2}{5}}$	$-\frac{1}{3}\sqrt{\frac{3}{5}}$	$\frac{2}{9}\sqrt{\frac{2}{5}}$	$\frac{1}{3\sqrt{5}}$	*
$N(^2D_M)_{\frac{5}{2}^+}$	$-\frac{1}{3}\sqrt{\frac{3}{5}}$	$-\frac{1}{3}\sqrt{\frac{2}{5}}$	$-\frac{2}{9}\sqrt{\frac{3}{5}}$	$-\frac{2}{3\sqrt{5}}$	*
$N(^4D_M)_{\frac{1}{2}^+}$	0	0	$\frac{1}{18\sqrt{5}}$	*	*
$N(^4D_M)_{\frac{3}{2}^+}$	0	0	$-\frac{1}{18\sqrt{5}}$	0	$\frac{1}{6\sqrt{15}}$
$N(^4D_M)_{\frac{5}{2}^+}$	0	0	$-\frac{1}{18}\sqrt{\frac{3}{35}}$	0	$-\frac{1}{6}\sqrt{\frac{6}{35}}$
$N(^4D_M)_{\frac{7}{2}^+}$	0	0	$\frac{1}{3\sqrt{70}}$	0	$\frac{1}{3\sqrt{42}}$
$\Delta(^2D_M)_{\frac{3}{2}^+}$	$-\frac{1}{3}\sqrt{\frac{2}{5}}$	$\frac{1}{3}\sqrt{\frac{3}{5}}$	$\frac{1}{9}\sqrt{\frac{2}{5}}$	$-\frac{1}{3\sqrt{5}}$	*
$\Delta(^2D_M)_{\frac{5}{2}^+}$	$\frac{1}{3}\sqrt{\frac{3}{5}}$	$\frac{1}{3}\sqrt{\frac{2}{5}}$	$-\frac{1}{9}\sqrt{\frac{3}{5}}$	$\frac{2}{3\sqrt{5}}$	*

Table 6: The angular momentum and flavor parts of the helicity amplitudes for $\gamma p \to \rho^+ n$ in the $SU(6)\otimes O(3)$ symmetry limit, while those for $\gamma n \to \rho^- p$ are given by $A(\gamma n \to \rho^- p) = (-1)^{I+1/2} A(\gamma p \to \rho^+ n)$, where I is the isospin of the resonances.

States	$S_{1/2}$	$A_{1/2}$		$A_{3/2}$	
	S	A	B	A	B
$N(^2P_M)_{\frac{1}{2}^-}$	$-\frac{2}{3\sqrt{6}}$	$\frac{2}{3\sqrt{3}}$	$-\frac{4}{9\sqrt{6}}$	$*$	$*$
$N(^2P_M)_{\frac{3}{2}^-}$	$\frac{2}{3\sqrt{3}}$	$\frac{2}{3\sqrt{6}}$	$\frac{4}{9\sqrt{3}}$	$\frac{2}{3\sqrt{2}}$	$*$
$N(^4P_M)_{\frac{1}{2}^-}$	0	0	$\frac{1}{9\sqrt{6}}$	$*$	$*$
$N(^4P_M)_{\frac{3}{2}^-}$	0	0	$-\frac{1}{9\sqrt{30}}$	0	$-\frac{1}{3\sqrt{10}}$
$N(^4P_M)_{\frac{5}{2}^-}$	0	0	$-\frac{1}{3\sqrt{30}}$	0	$-\frac{1}{3\sqrt{15}}$
$\Delta(^2P_M)_{\frac{1}{2}^-}$	$-\frac{1}{3\sqrt{6}}$	$\frac{1}{3\sqrt{3}}$	$\frac{1}{9\sqrt{6}}$	$*$	$*$
$\Delta(^2P_M)_{\frac{3}{2}^-}$	$\frac{1}{3\sqrt{3}}$	$\frac{1}{3\sqrt{6}}$	$-\frac{1}{9\sqrt{3}}$	$\frac{1}{3\sqrt{2}}$	$*$
$N(^2D_S)_{\frac{3}{2}^+}$	$-\frac{1}{3}\sqrt{\frac{2}{5}}$	$\frac{1}{3}\sqrt{\frac{3}{5}}$	$-\frac{5}{9}\sqrt{\frac{2}{5}}$	$-\frac{1}{3\sqrt{5}}$	$*$
$N(^2D_S)_{\frac{5}{2}^+}$	$\frac{1}{3}\sqrt{\frac{3}{5}}$	$\frac{1}{3}\sqrt{\frac{2}{5}}$	$\frac{5}{9}\sqrt{\frac{3}{5}}$	$\frac{2}{3\sqrt{5}}$	$*$
$\Delta(^4D_S)_{\frac{1}{2}^+}$	0	0	$\frac{2}{9\sqrt{10}}$	$*$	$*$
$\Delta(^4D_S)_{\frac{3}{2}^+}$	0	0	$-\frac{2}{9\sqrt{10}}$	0	$\frac{1}{3}\sqrt{\frac{2}{15}}$
$\Delta(^4D_S)_{\frac{5}{2}^+}$	0	0	$-\frac{1}{9}\sqrt{\frac{6}{35}}$	0	$\frac{2}{3}\sqrt{\frac{3}{35}}$
$\Delta(^4D_S)_{\frac{7}{2}^+}$	0	0	$\frac{2}{3\sqrt{35}}$	0	$\frac{2}{3\sqrt{21}}$
$N(^2S'_S)_{\frac{1}{2}^+}$	$\frac{1}{3}$	$*$	$\frac{5}{9}$	$*$	$*$
$\Delta(^4S'_S)_{\frac{3}{2}^+}$	0	$*$	$\frac{2}{9\sqrt{2}}$	$*$	$\frac{1}{3}\sqrt{\frac{2}{3}}$
$\Delta(^4S_S)_{\frac{3}{2}^+}$	0	$*$	$\frac{2}{9\sqrt{2}}$	$*$	$\frac{1}{3}\sqrt{\frac{2}{3}}$
$N(^2S_M)_{\frac{1}{2}^+}$	$\frac{2}{3\sqrt{2}}$	$*$	$\frac{4}{9\sqrt{2}}$	$*$	$*$
$N(^4S_M)_{\frac{3}{2}^+}$	0	$*$	$-\frac{1}{9\sqrt{2}}$	$*$	$-\frac{1}{3\sqrt{6}}$
$\Delta(^2S_M)_{\frac{1}{2}^+}$	$\frac{1}{3\sqrt{2}}$	$*$	$-\frac{1}{9\sqrt{2}}$	$*$	$*$
$N(^2D_M)_{\frac{3}{2}^+}$	$-\frac{2}{3}\sqrt{\frac{2}{5}}$	$\sqrt{\frac{2}{15}}$	$-\frac{4}{9\sqrt{5}}$	$-\frac{1}{3}\sqrt{\frac{2}{5}}$	$*$
$N(^2D_M)_{\frac{5}{2}^+}$	$\sqrt{\frac{2}{15}}$	$\frac{2}{3\sqrt{5}}$	$\frac{4}{9}\sqrt{\frac{3}{10}}$	$\frac{2}{3}\sqrt{\frac{2}{5}}$	$*$
$N(^4D_M)_{\frac{1}{2}^+}$	0	0	$-\frac{1}{9\sqrt{10}}$	$*$	$*$
$N(^4D_M)_{\frac{3}{2}^+}$	0	0	$\frac{1}{9\sqrt{10}}$	0	$-\frac{1}{3\sqrt{30}}$
$N(^4D_M)_{\frac{5}{2}^+}$	0	0	$\frac{1}{9}\sqrt{\frac{3}{70}}$	0	$\frac{1}{3}\sqrt{\frac{3}{35}}$
$N(^4D_M)_{\frac{7}{2}^+}$	0	0	$-\frac{1}{3\sqrt{35}}$	0	$-\frac{1}{3\sqrt{21}}$
$\Delta(^2D_M)_{\frac{3}{2}^+}$	$-\frac{1}{3\sqrt{5}}$	$\frac{1}{\sqrt{30}}$	$\frac{1}{9\sqrt{5}}$	$-\frac{1}{3\sqrt{10}}$	$*$
$\Delta(^2D_M)_{\frac{5}{2}^+}$	$\frac{1}{\sqrt{30}}$	$\frac{1}{3\sqrt{5}}$	$-\frac{1}{9}\sqrt{\frac{3}{10}}$	$\frac{1}{3}\sqrt{\frac{2}{5}}$	$*$

Table 7: The spatial integrals in the harmonic oscillator basis.

Multiplet	Expression
$[70,1^-]_1$	$A = \frac{3a}{2m_q\sqrt{3}}\alpha exp(-\frac{\mathbf{q}^2}{6\alpha^2})$
	$B = \frac{b'}{m_q}\sqrt{\frac{3}{2}}\frac{\mathbf{q}^2}{\alpha}exp(-\frac{\mathbf{q}^2}{6\alpha^2})$
	$S = \frac{\sqrt{3}\mu a}{\alpha}exp(-\frac{\mathbf{q}^2}{6\alpha^2})$
$[56,2^+]_2$	$A = -\frac{a}{2\sqrt{2}m_q}\lvert\mathbf{q}\rvert exp(-\frac{\mathbf{q}^2}{6\alpha^2})$
	$B = -\frac{b'}{2\sqrt{3}m_q}\lvert\mathbf{q}\rvert(\frac{\mathbf{q}}{\alpha})^2 exp(-\frac{\mathbf{q}^2}{6\alpha^2})$
	$S = -\frac{\mu a}{\sqrt{6}\lvert\mathbf{q}\rvert}(\frac{\mathbf{q}}{\alpha})^2 exp(-\frac{\mathbf{q}^2}{6\alpha^2})$
$[56,0^+]_2$	$B = \frac{b'}{2\sqrt{3}m_q}\lvert\mathbf{q}\rvert(\frac{\mathbf{q}}{\alpha})^2 exp(-\frac{\mathbf{q}^2}{6\alpha^2})$
	$S = \frac{\mu a}{\sqrt{6}\lvert\mathbf{q}\rvert}(\frac{\mathbf{q}}{\alpha})^2 exp(-\frac{\mathbf{q}^2}{6\alpha^2})$
$[56,0^+]_0$	$B = \frac{3b'}{\sqrt{2}m_q}\lvert\mathbf{q}\rvert exp(-\frac{\mathbf{q}^2}{6\alpha^2})$
	$S = \frac{3\mu a}{\lvert\mathbf{q}\rvert}exp(-\frac{\mathbf{q}^2}{6\alpha^2})$
$[70,0^+]_2$	$B = -\frac{b'}{2\sqrt{6}m_q}\lvert\mathbf{q}\rvert(\frac{\mathbf{q}}{\alpha})^2 exp(-\frac{\mathbf{q}^2}{6\alpha^2})$
	$S = -\frac{\mu a}{2\sqrt{3}\lvert\mathbf{q}\rvert}(\frac{\mathbf{q}}{\alpha})^2 exp(-\frac{\mathbf{q}^2}{6\alpha^2})$
$[70,2^+]_2$	$A = \frac{a}{2\sqrt{2}m_q}\lvert\mathbf{q}\rvert exp(-\frac{\mathbf{q}^2}{6\alpha^2})$
	$B = \frac{b'}{2\sqrt{3}m_q}\lvert\mathbf{q}\rvert(\frac{\mathbf{q}}{\alpha})^2 exp(-\frac{\mathbf{q}^2}{6\alpha^2})$
	$S = \frac{\mu a}{\sqrt{6}\lvert\mathbf{q}\rvert}(\frac{\mathbf{q}}{\alpha})^2 exp(-\frac{\mathbf{q}^2}{6\alpha^2})$